Homework (analysis): set 1.

1. The following formula says that *disjunction is distributive over conjunction*. Writing a truth table show that it is a tautology (that means it is true for all possible valuations):

$$p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r).$$

2. Prove by induction:

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \cdots + \frac{1}{n \cdot (n+1)} = \frac{n}{n+1}.$$

3. Solve:

$$\frac{\sqrt{2x-1}}{x-2} < 1.$$

4. A nuclear reactor has just had a serious radiation leak and the danger zone is described as the area within 50 km of the reactor R. The Prime Minister plans to fly a helicopter from her home H to destination D. The reactor is 60 km east and 30 km north of the Prime Minister's home H. Destination D is 50 km east and 150 km north of H. If the helicopter is flown in a straight line from H to D, will it cross the danger zone?
Hint: Set up a coordinate system with H located at the origin.

5. Find the domain and the range of functions:

$$f(x) = \frac{x^3 - 1}{x - 1}, \qquad g(x) = \log_3(1 + |x|).$$

Please write the solutions clearly (by hand) on A4 paper and give it to me on (or before) 30/10/2018.
Every solution will be given 1 point (correct, minor error possible), 0.5 pt. (good idea, but not all correct), 0 pt. (nothing worthy). The maximum for this homework is 5 pts.