Problems for the exam (analysis): series.

1. Find the partial sum s_n of the series

a) $\sum\limits_{n=1}^{\infty} \frac{300}{2^n}$;

b) $\sum\limits_{n=1}^{\infty} \ln(1+\frac{1}{n})$;

c) $\sum\limits_{n=1}^{\infty} \frac{2^n}{300}$;

Is the series convergent?

2. Determine whether the series is convergent.

a) $\sum\limits_{n=1}^{\infty} \frac{n^3}{2^n}$;

b) $\sum\limits_{n=1}^{\infty} \frac{\sqrt{n+1}-\sqrt{n}}{n}$.

c) $\sum\limits_{n=1}^{\infty} \frac{(n!)^2 \cdot 5^n}{(2n)!}$;

d) $\sum\limits_{n=1}^{\infty} \frac{1}{\sqrt{n}} \sin\frac{1}{n}$;

e) $\sum\limits_{n=1}^{\infty} \frac{n^n}{n!}$;

f) $\sum\limits_{n=1}^{\infty} \frac{n^{3n}}{(3n)!}$;

g) $\sum\limits_{n=1}^{\infty} \frac{1}{(n!)^2-n!}$.

3. Find the radius of convergence of power series

a) $\sum\limits_{n=0}^{\infty} \frac{x^n}{\sqrt{n}}$;

b) $\sum\limits_{n=0}^{\infty} \frac{3^n}{n+1} x^n$.

c) $\sum\limits_{n=1}^{\infty} \frac{n^2}{3^n} x^n$.

d) $\sum\limits_{n=1}^{\infty} \frac{n!}{(2n)!} x^n$.

4. Using the principal expansions

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + \cdots, \qquad |x| < 1$$

$$e^x = \sum_{n=0}^{\infty} \frac{1}{n!} x^n = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \cdots;$$

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots;$$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots.$$

find expansions in Maclaurin series for the functions:

a) $\frac{1}{2+x}$;

b) $\frac{1}{1-2x}$;

c) $\sin^2 x = \frac{1}{2}(1-\cos 2x)$;

d) xe^{-x};

e) $\frac{1}{1-x^2}$;

f) $x^2 \sin(2x)$.