

## Topics for the exam: algebra

**You should be able to explain, in writing, the following.**

1. Roots of polynomials. Factoring polynomials.
2. Greatest common divisor and Euclidean algorithm.
3. Concept of a complex number as a pair of real numbers.
4. Field of complex numbers as an extension of the field of real numbers.
5. Absolute value of a complex number. Geometric interpretation.
6. Argument of a complex number.
7. Polar form of a complex number.
8. The Moivre formula.
9. Exponential form of a complex number.
10. Roots of a complex number.
11. Fundamental theorem of algebra.
12. Concept of a matrix. Operations on matrices.
13. A power of a square matrix. Matrix polynomials.
14. Determinant of a matrix. Expansion in cofactors.
15. System of linear equations and its solution set.
16. Elementary row operations and row equivalent matrices.
17. Echelon (and reduced echelon) form of a matrix. Leading variables and free variables.
18. Gauss-Jordan elimination method.
19. The row reduced augmented matrix of a system determines the number of solutions of the system – describe how.
20. Cramer's Rule.
21. Vector space. Space  $\mathbb{R}^n$ .
22. Linear combination and linear independence of vectors. Basis for a vector space.
23. The span of a set of vectors.
24. Eigenvalues and eigenvectors of a matrix.
25. Diagonalization of a matrix.

**You should also be competent in the following skills:**

1. Perform operations on polynomials, especially divide them.
2. Find roots of polynomials and factor them.
3. Perform operations on complex numbers in rectangular form.
4. Interpret complex numbers on a plane.
5. Plot sets defined by complex equality or inequality.
6. Find polar form of a complex number.
7. Apply the Moivre formula.
8. Express a complex number in exponential form.
9. Determine square roots of a complex number in rectangular form.
10. Determine  $n$ th roots of a complex number in polar form.
11. Solve simple equations with complex coefficients.
12. Perform operations on matrices.
13. Determine powers and polynomials of a square matrix.
14. Find determinant of a square matrix.
15. Find the inverse of a square nonsingular matrix.
16. Solve systems of linear equations using various methods (elimination, Cramer's Rule, matrix inverse).
17. Find linear combinations of vectors and check linear independence of a set of vectors.
18. Determine eigenvectors and eigenvalues of a matrix.