

TRIGONOMETRIC EQUATIONS

Remember:

Degrees	30°	45°	60°	90°	120°	135°	150°	180°	270°	360°
Radians	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{3\pi}{2}$	2π

x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin x$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos x$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\operatorname{tg} x$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	X
$\operatorname{ctg} x$	X	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0

You should understand *reduction formulas of trigonometry*, such as

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x$$

$$\sin(\pi - x) = \sin x$$

$$\cos\left(\frac{\pi}{2} - x\right) = \sin x$$

$$\cos(\pi - x) = -\cos x$$

$$\operatorname{tg}\left(\frac{\pi}{2} - x\right) = \operatorname{ctg} x$$

Solutions of elementary equations:

$$\sin x = a \implies x = x_0 + 2k\pi \text{ or } x = \pi - x_0 + 2k\pi, \text{ where } x_0 \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

$$\cos x = a \implies x = x_0 + 2k\pi \text{ or } x = -x_0 + 2k\pi, \text{ where } x_0 \in [0, \pi]$$

$$\tan x = a \implies x = x_0 + k\pi, \text{ where } x_0 \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

$$\operatorname{ctg} x = a \implies x = x_0 + k\pi, \text{ where } x_0 \in (0, \pi)$$

Solve equations and inequalities:

1. $\sin x + \cos x = 1$

2. $\sin^2 x = \sin x$

3. $4 \sin^2 x + \sin^2 2x = 3$

4. $\operatorname{ctg} x - \cos x = \frac{1 - \sin x}{2 \sin x}$

5. $\cos^2 x < \frac{1}{2}$

6. $\sin x > \cos x$

7. $\cos x + \operatorname{tg} x < 1 + \sin x, 0 < x < 2\pi$

8. $2 \sin^2 3x + \sin^2 5x < 2$

9. $|\sin 3x| - |\cos 3x| = 1$

10. $|\sin x| > \frac{\sqrt{3}}{2}$